

Determining Anisotropic Transmissivity Using a Simplified Papadopoulos Method

by Victor M. Heilweil¹ and Paul A. Hsieh²

Abstract

The straight-line method presented by Papadopoulos requires a minimum of three observation wells for determining the transmissivity tensor of a homogeneous and anisotropic aquifer. A simplification of this method was developed for fractured aquifers where the principal directions of the transmissivity tensor are known prior to implementation, such as when fracture patterns on outcropping portions of the aquifer may be used to infer the principal directions. This new method assumes that observation wells are drilled along the two principal directions from the pumped well, thus reducing the required number of observation wells to two. This method was applied for an aquifer test in the fractured Navajo Sandstone of southwestern Utah and yielded minimum and maximum principal transmissivity values of 70 and 1800 m²/d, respectively, indicating an anisotropy ratio of ~24 to 1.

Introduction

Papadopoulos (1965) presented a method that can be used to determine the transmissivity tensor for a homogeneous and anisotropic aquifer of infinite areal extent based on the analyses of observation well data from a constant-pumping rate aquifer test. Three or more observation wells at different directions from the pumping well are necessary. However, if the principal directions of the transmissivity tensor can be assumed from prior geologic information, then only two wells located along these two directions are necessary. This may be the situation in an aquifer with orthogonally oriented fractures where detailed surface fracture mapping and/or borehole data from the aquifer test site are available. If the observation wells are drilled along these two principal axes, the straight-line method presented by Papadopoulos (1965) can be applied to late-time drawdown or recovery data

from two observation wells that display similar slopes on a semilog plot. In field situations where observed fracture sets are not perpendicular to each other, it is more difficult to estimate the principal axes of the transmissivity tensor and this method would have less utility.

Theory

Assuming that observation wells A and B are located along the maximum and minimum principal axes directions in an orthogonal orientation with respect to each other and the pumping well, the drawdown in the observation wells in a homogeneous and anisotropic aquifer of infinite areal extent is given by Papadopoulos (1965, equations 15 and 16):

$$s(x, y, t) = \frac{Q}{4\pi\sqrt{T_{xx}T_{yy}}} W(u_{xy}) \quad (1)$$

with

$$u_{xy} = \frac{S}{4t} \left(\frac{T_{xx}y^2 + T_{yy}x^2}{T_{xx}T_{yy}} \right) \quad (2)$$

where s is drawdown, t is time, Q is pumping rate, $W(u_{xy})$ is the well function of u_{xy} , T_{xx} and T_{yy} are the transmissivities along principal axes, and S is aquifer storage. Note that x and y represent ξ and η as described by Papadopoulos (1965).

¹Corresponding author: U.S. Geological Survey, 2329 Orton Circle, Salt Lake City, UT 84119; (801) 908-5042; fax (801) 908-5001; heilweil@usgs.gov

²U.S. Geological Survey, Building 15, McKelvey Building, Mail Stop 496, Menlo Park, CA 94025; (650) 329-4580; fax (650) 329-4327, pahsieh@usgs.gov

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Applying the preceding solution to observation well A, which is located at $x = x_A, y = 0$ yields:

$$s(x_A, 0, t) = \frac{Q}{4\pi\sqrt{T_{xx}T_{yy}}} W\left(\frac{Sx_A^2}{4T_{xx}t}\right) \quad (3)$$

In comparing this equation to the Theis solution (Theis 1935)

$$s(r, t) = \frac{Q}{4\pi T} W\left(\frac{Sr^2}{4Tt}\right) \quad (4)$$

the analogies are as follows: T of the Theis solution is substituted with $\sqrt{T_{xx}T_{yy}}$, S/T of the Theis solution is substituted with S/T_{xx} , and r of the Theis solution is substituted with x_A . This analogy can be extended to the Cooper-Jacob straight-line method (Cooper and Jacob 1946), which also can be modified for anisotropic conditions. Under ideal conditions in a homogeneous anisotropic aquifer, Papadopoulos shows that the straight-line parts of all observation well data on a semilog plot should have the same slope, so that the intercepts would yield T_{xx} and T_{yy} (Figure 1). In the Cooper-Jacob method, the slope of the late-time (straight-line part) data yields transmissivity from the determination of the change in draw-down (Δs) per log cycle (Δt), and the intercept gives S/T and thus S . Substituting $\sqrt{T_{xx}T_{yy}}$ for T yields the following equation modified from the Cooper-Jacob method:

$$\sqrt{T_{xx}T_{yy}} = \frac{2.30Q}{4\pi\Delta s} \quad (5)$$

for T in m^2/d , Q in m^3/min , and Δs in m . Likewise, substituting S/T_{xx} for S/T yields the following equation modified from the Cooper-Jacob method:

$$\frac{S}{T_{xx}} = \frac{2.25t_{0a}}{[x_A]^2} \quad (6)$$

where t_{0a} is the x-intercept (time) for well A, and x_A is the distance from the pumped well to well A. Note that the coefficient of 2.25 is obtained by setting the well function, $W(u_{xy})$, to zero and combining terms within the log

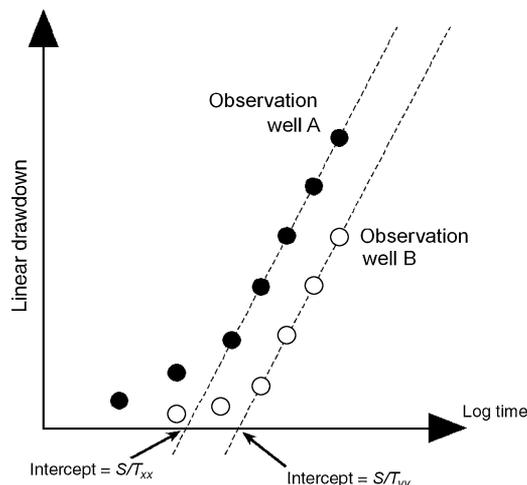


Figure 1. Idealized data set for an anisotropic and homogeneous aquifer.

function (4 from u and $1/1.781$ from the Taylor series remainder of -0.5772).

Next, applying the anisotropic solution to observation well B, which is located at $x = 0, y = y_B$, yields:

$$s(0, y_B, t) = \frac{Q}{4\pi\sqrt{T_{xx}T_{yy}}} W\left[\frac{Sy_B^2}{4T_{yy}t}\right] \quad (7)$$

where $\sqrt{T_{xx}T_{yy}}$ is like T of Theis, S/T_{yy} is like S/T of Theis, and y_B is like r of Theis. After plotting the data from observation well B on semilog paper, the straight-line parts fitted to the data must have the same slope (and Δs) as the observation well A data set. This ensures that $T_{xx}T_{yy}$ computed from observation well B data is equal to $T_{xx}T_{yy}$ computed from observation well A data. By the same reasoning, substituting T_{yy}/S for T/S yields the following equation modified from the Cooper-Jacob method (Cooper and Jacob 1946, Equation 8):

$$\frac{S}{T_{yy}} = \frac{2.25t_{0b}}{[y_B]^2} \quad (8)$$

where t_{0b} is the x-intercept (time) for well B and y_B is the distance from the pumped well to well B.

In summary, the previous straight-line fits to the observation well A and B data sets on a semilog graph should yield $\sqrt{T_{xx}T_{yy}}$ (the value from each of the data sets should be the same), S/T_{xx} , and S/T_{yy} . The following procedure can be used to determine T_{xx} , T_{yy} , and S separately:

1. Square $\sqrt{T_{xx}T_{yy}}$ to obtain $T_{xx}T_{yy}$
2. Multiply S/T_{xx} and S/T_{yy} to obtain $S^2/(T_{xx}T_{yy})$
3. Multiply the result from steps 1 and 2 to get S^2 , then take the square root to arrive at S
4. Divide the S obtained from step 3 by S/T_{xx} to get T_{xx}
5. Divide the S obtained from step 3 by S/T_{yy} to get T_{yy} .

T_{xx} is known as the “principal transmissivity in the direction of the x-axis.” T_{yy} is known as the “principal transmissivity in the direction of the y-axis.” If $T_{xx} > T_{yy}$, then the x-axis points along the major principal direction and the y-axis points along the minor principal direction. If $T_{yy} > T_{xx}$, then the y-axis points along the major principal direction and the x-axis points along the minor principal direction. Therefore, it is not necessary (nor warranted) to assume which is the major and which is the minor principal direction at the start of the analysis.

Application

A multiple-observation well aquifer test was conducted in the Navajo Aquifer at Anderson Junction in southwestern Utah (Figure 2) to determine the transmissivity and storage properties and evaluate anisotropy. Because of the uniformity of the well-sorted eolian sandstone, it is assumed that all anisotropy is caused by secondary permeability associated with secondary fracturing. The aquifer test site is in a highly fractured region of outcropping sandstone that has two predominant clusters of fracturing at orientations of 180° to 210° and 90° to 130° (Figure 3). On the basis of this fracture study (Hurlow 1998), two observation wells were drilled to a depth of

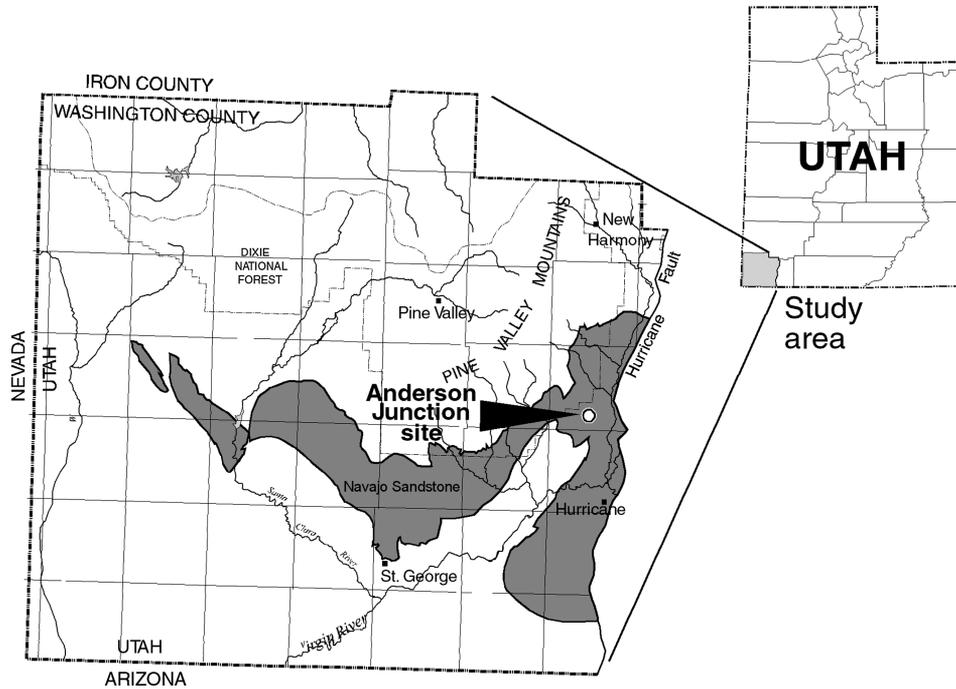


Figure 2. Location of the Anderson Junction aquifer test, Washington County, Utah.

~120 m specifically for the aquifer test at approximately the same radial distance from the production well but at perpendicular orientations. The total depth of the production well is ~180 m, with casing set to 150 m. The drillers logs for all three wells indicate uniform fine-grained sandstone beneath 1 to 12 m of unconsolidated soil. Observation well A is located 117 m east-southeast of the production well along a 110° orientation (parallel to the 90° to 130° azimuthal cluster of fractures). The static water level in well A before pumping was 6.4 m.

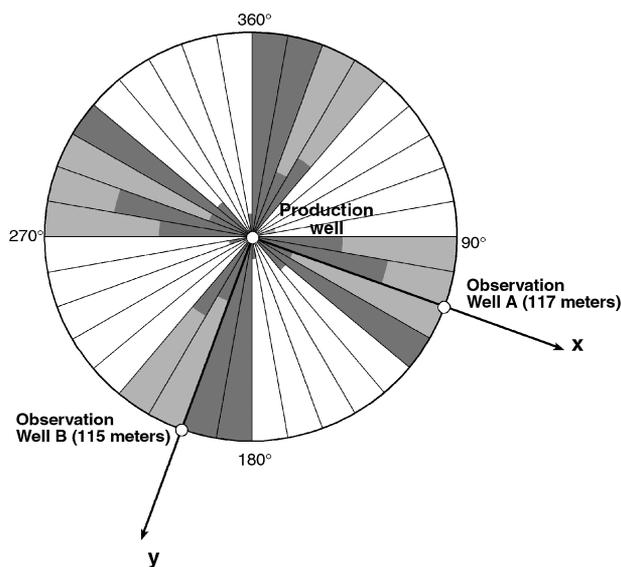


Figure 3. Rose diagram showing fracture orientations (from Hurlow 1998) and locations of production and monitoring wells used for the Anderson Junction aquifer test, Washington County, Utah.

Observation well B is located 115 m south-southwest of the production well along a 200° orientation (parallel to the 180° to 210° azimuthal cluster of fractures). The static water level in well B before pumping was 9.4 m. A simplifying assumption was made that the orientation of the fracturing within the aquifer is the same as that of the surface fractures. This assumption is justified by both regional aerial photos showing the uniform direction of these fracture lineaments and cross-sectional observations of the planar nature of the fractures throughout the entire exposed 2000-foot thickness of the Navajo Sandstone at nearby Zion National Park and Snow Canyon State Park.

The multiple-well aquifer test involved pumping the production well for ~4 d at an average rate of 4.2 m³/min. Discharge was measured with a pito tube, v-notch weir, and pygmy meter. The discharge from the production well was diverted into a 0.38-m-diameter ABS drain pipe, which transported the water 150 m away from the well to a natural dry wash. In addition to the two observation wells, a preexisting well (the “original” well) located 3 m due east of the production well also was used for evaluating drawdown. Water levels were measured in the three observation wells and the production well for 4 d prior to the test, during the 4 d of pumping and for as many as 20 d after the pump was shut off.

Measured water levels at the observation wells were not corrected for barometric changes because the magnitude of drawdown and recovery at all the wells was much larger (5.8 to 24.4 m) than the effects of barometric changes (generally <0.3 m). Prepumping trend corrections were applied to all the observation well drawdown data because of a rise in water levels resulting from recovery after the development of the production well shortly before the aquifer test. Precovery trend corrections were applied to the observation well recovery data

because water levels did not reach a pumping equilibrium before the production well was shut off.

The recovery data for the three observation wells were initially plotted together on log-log scale by dividing time by the observation well's radial distance squared. The recovery data from the closest observation well (original well) were eliminated from the analysis because of the delayed response in early-time data caused by wellbore storage effects resulting from proximity to the pumped well and large borehole diameter (15 cm). Also, the maximum recovery at this observation well (as much as 24.4 m) made up a substantial part of the saturated thickness of the aquifer, resulting in a substantial change in transmissivity during the aquifer test.

The data sets from the remaining two observation wells (A and B) were initially analyzed with three curve-matching solutions: (1) the Theis (1935) solution for confined aquifers; (2) the modified Hantush (Lohman 1972) solution for leaky confined aquifers with vertical movement; and (3) the Neuman (1974) solution for unconfined aquifers with delayed yield. None of these type curves fit both sets of data, indicating that these methods were not applicable for interpreting these results. In particular, the assumption of isotropy in the three methods is not valid. The presence of anisotropy at the Anderson Junction test site is indicated by the large difference in observed drawdown at the two observation wells: 10.1 m drawdown at observation well A aligned with the 110° fracture orientation, compared with 5.8 m drawdown at observation well B aligned with the 200° fracture orientation. These observations are consistent with a fractured anisotropic aquifer.

Therefore, the simplified version of the Papadopoulos (1965) method for data analysis from a homogeneous and anisotropic aquifer was used, assuming that the two observation wells are parallel to the two principal axes.

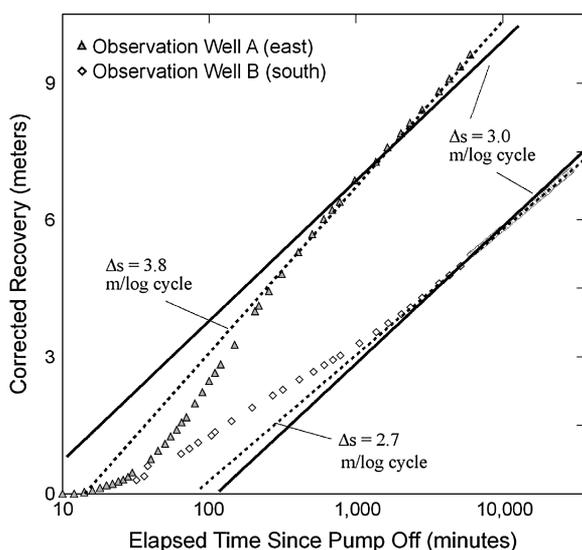


Figure 4. Semilog plot of recovery data from the Anderson Junction aquifer test with the solid fitted lines having equal slopes (Δs) of 3 m recovery per log cycle and the dashed fitted lines having slopes ranging from 2.7 to 3.8 m recovery per log cycle.

The corrected recovery data for both observation wells were plotted on a semilog graph. In an ideal homogeneous anisotropic aquifer, the slopes of observation well data sets should be the same. However, unlike the ideal case, the slopes of the straight-line parts of the two observation well data sets for this aquifer test are not identical (Figure 4). With these two unequal slopes, the square root of $T_{xx}T_{yy}$ computed from observation well A does not equal that computed from observation well B. This indicates that the aquifer is not completely homogeneous at this site. Nonetheless, because the late-time data of each plot are similar and approach straight lines, the same slope (Δs) of 3 m of drawdown per log cycle was fitted to each data set. By forcing both lines to have the same slope, the product $T_{xx}T_{yy}$ from both wells is the same and the data can be interpreted by using a homogeneous anisotropic aquifer model.

Substituting these values into the Cooper-Jacob equation (Equation 5), with $Q = 4.2 \text{ m}^3/\text{min}$, yields the relation:

$$\sqrt{T_{xx}T_{yy}} = 360 \text{ m}^2/\text{d} \quad (9)$$

Also from Figure 4, the x-intercept on the semilog plot for the well A recovery data is 5.5 min (0.0038 d); the x-intercept on the semilog plot for the well B recovery data is 110.0 min (0.0764 d). Substituting the distance to well A (x_A) of 117 m and the distance to well B (y_B) of 115 m into Equations 6 and 8 yields:

$$\frac{S}{T_{xx}} = 5.99 \times 10^{-7} \quad \frac{S}{T_{yy}} = 1.19 \times 10^{-5} \quad (10)$$

Solving these three relations simultaneously yields $T_{xx} \cong 1600 \text{ m}^2/\text{d}$, $T_{yy} \cong 80 \text{ m}^2/\text{d}$, and $S \cong 0.001$.

Heterogeneities within the Navajo Aquifer at Anderson Junction do not permit a unique equal-slope fit to the semilog plot of observation well data from wells A and B; therefore, an analysis of the possible range of values is necessary. To determine the maximum amount of interpretative error that may be introduced by "forcing" lines of equal slope to both observation well data sets, the steepest and shallowest possible fitted slopes are also shown in Figure 4. The steepest possible slope for the two data sets corresponds to the best fit for the well A data set. The shallowest possible slope for the two data sets corresponds to the best fit for the well B data set. On the basis of these alternative slopes and x-intercepts, values range for T_{xx} from 1400 to 2100 m^2/d , for T_{yy} from 60 to 80 m^2/d , and for S from 0.0007 to 0.0025. Therefore, the average of the maximum and minimum possible values for the transmissivity and storage from the Anderson Junction aquifer test, including error brackets, is $T_{xx} \cong 1800 \text{ m}^2/\text{d} \pm 21\%$, $T_{yy} \cong 70 \text{ m}^2/\text{d} \pm 19\%$, and $S \cong 0.0013 \pm 1/4 \text{ log cycle}$. This indicates that the ratio of transmissivity (anisotropy factor) in the 110° and 200° orientations is ~24:1, but could range from 23:1 to 25:1, depending on the fitted slope. With an assumed aquifer thickness of 180 m, horizontal hydraulic conductivity ranges from ~0.4 m/d in the 200° orientation to 9.8 m/d in the 110° orientation. These values are based on the assumption that the two observation wells are located along the principal directions of the transmissivity/

hydraulic conductivity tensor. If this assumption is incorrect, the range of transmissivity and hydraulic conductivity values, as well as anisotropy ratios, would be even larger. However, without a third observation well, this potential error cannot be evaluated.

The range of hydraulic conductivity values determined from this aquifer test analysis is generally greater than values from aquifer testing and laboratory core testing from other nearby sites in the Navajo Sandstone. Aquifer tests in other parts of the Navajo Sandstone with less prominent surface fracturing produced hydraulic conductivity values ranging from 0.06 to 0.7 m/d (Heilweil et al. 2000). Laboratory-saturated hydraulic conductivity measurements ranged from 0.01 to 0.42 m/d in the nearby Hurricane Bench area (Heilweil et al. 2004) and 0.11 to 1.5 m/d elsewhere in southwestern Utah (Cordova 1978). However, the laboratory-determined values do not include the effects of open fractures that would increase the actual in situ hydraulic conductivity. Therefore, the Anderson Junction aquifer test data may indicate that along the minor principal direction (200° orientation), the hydraulic conductivity value of 0.4 m/d is characteristic of unfractured rock and that the fractures along this orientation might be closed or unconnected. In the major principal direction (110° orientation), the hydraulic conductivity value of 9.8 m/d is ~1 order of magnitude higher than the range of laboratory values, indicating that fractures along this orientation might be open and more hydraulically connected.

Conclusions

For aquifer testing of homogeneous and anisotropic aquifers having orthogonally oriented fracture directions assumed to align with the maximum and minimum principal axes of transmissivity, the simplified Papadopulos method only requires two observation wells, rather than the three observation wells required by the original Papadopulos method. This assumption is likely valid for aquifers having uniform fracture orientations, which may be discernible from either plan view and cross-sectional outcrop observations or borehole geophysical logging. In these special situations, the simplified Papadopulos method can reduce the drilling costs of observation wells, while still solving for the transmissivity tensor. The application of this simplified method for a multiple-well aquifer test of the Navajo Sandstone in southwestern Utah yields transmissivity values ranging from 70 to 1800 m²/d, indicating an anisotropy ratio of ~24:1. The required

forcing of equal slope for fitting data from both observation wells A and B of the Anderson Junction aquifer test introduced an uncertainty of ~20% for this data set, likely due to slight aquifer heterogeneity. The interpretive error introduced at other field sites through “forcing” lines of equal slope for the two observation well data sets will be dependent on the scale of a particular aquifer’s heterogeneity compared to the scale of the field test.

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